Vectors

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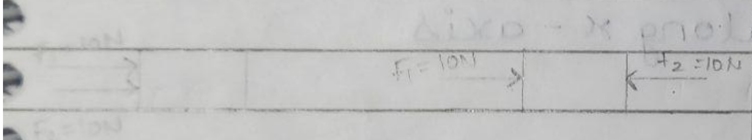
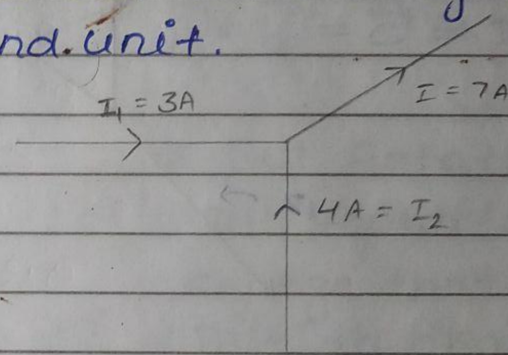
Vector

Vector	Scalar
Ex- force, velocity, acceleration, displacement, Electric field.	Ex- Mass, Temperature, Speed, distance, Current, pressure, density, flux, volume
# A Physical Quantity which have magnitude, direction, unit and must follow triangle law of vector addition	# A physical Quantity which have magnitude and unit.





$F_{net} = 20N$, $F_{net} = 0N$

* Vector Representation (Mathematically)

$\vec{A} = A\hat{A}$ ← direction only

↑ vector ↑ Magnitude

$\vec{F}$ = force

$F = |\vec{F}|$ = Magnitude of force

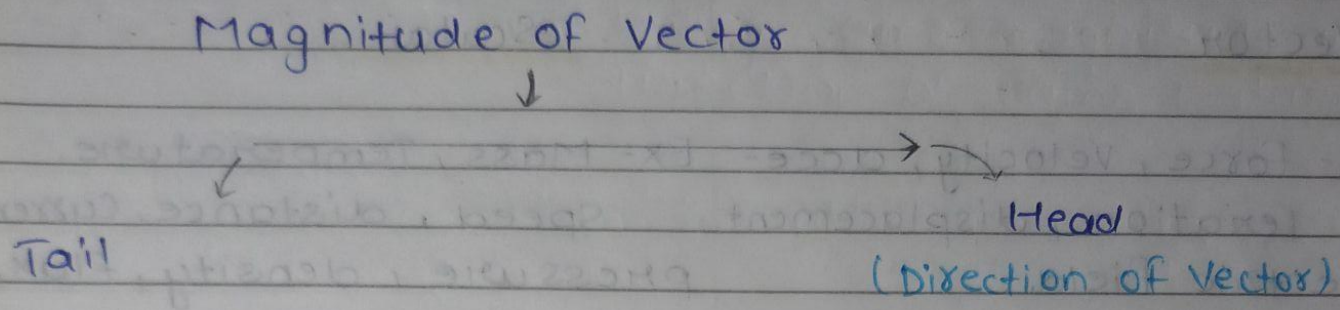
$\vec{F} = F\hat{F} \rightarrow \hat{F} = \frac{\vec{F}}{F}$

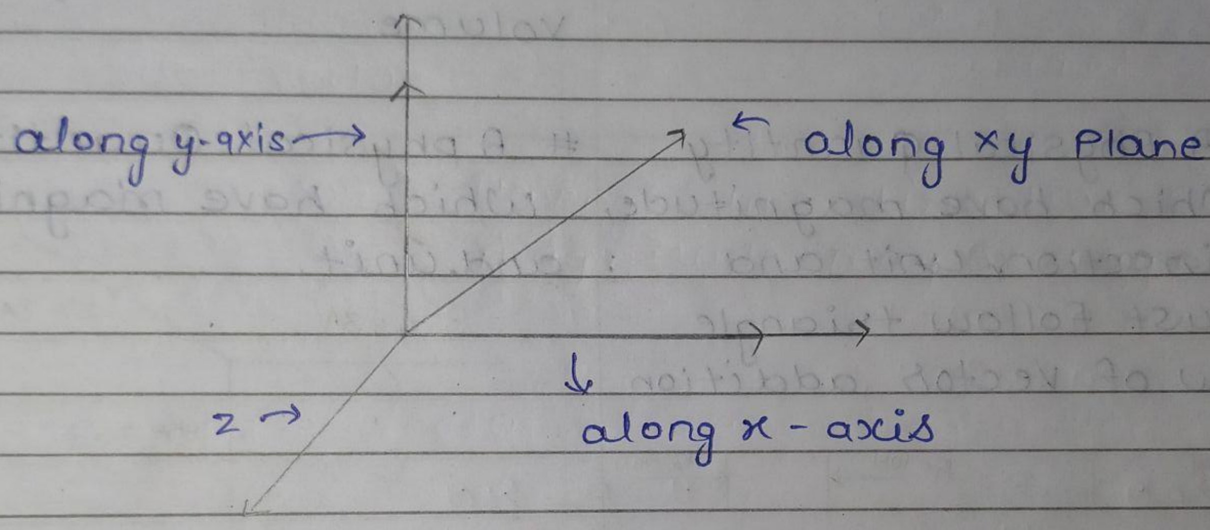
→ Unit vector of force

→ Represent direction of vector

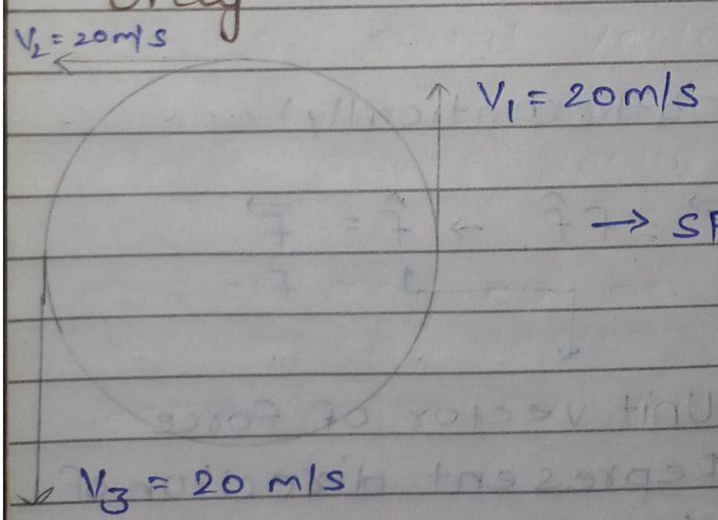
$\hat{F}$ → only have direction and have magnitude (one)

→ (Graphically)





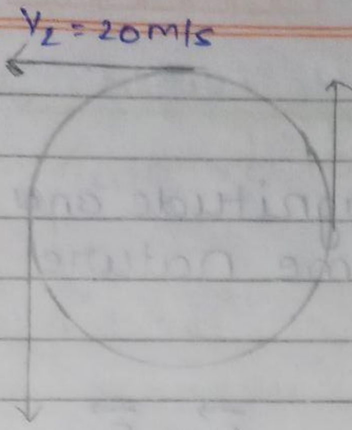
* Scalar can be change by changing magnitude only



→ speed remains same

$V_1 = V_2 = V_3$ (speed)

* Vector can be change by changing magnitude as well as direction.



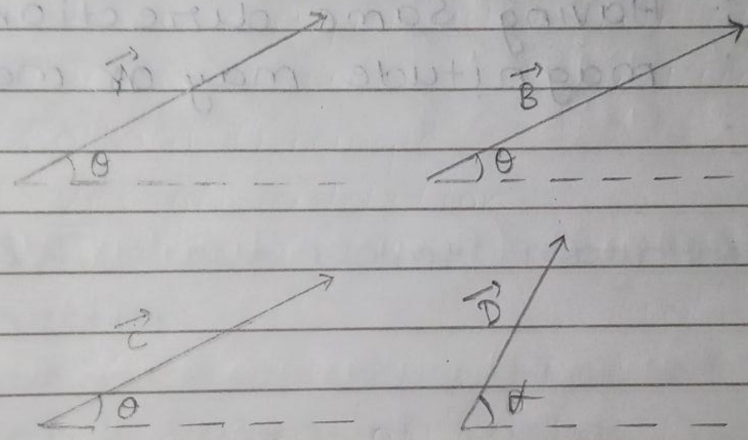
$\vec{V_1} \neq \vec{V_2}$

$|\vec{V_1}| = |\vec{V_2}|$

$O \rightarrow \vec{V_1} = 10 m/s$ $O \rightarrow \vec{V_2} = 20 m/s$

$\vec{V_1} \neq \vec{V_2}$

Vector can be shifted Parallel to itself by keeping magnitude and direction fixed

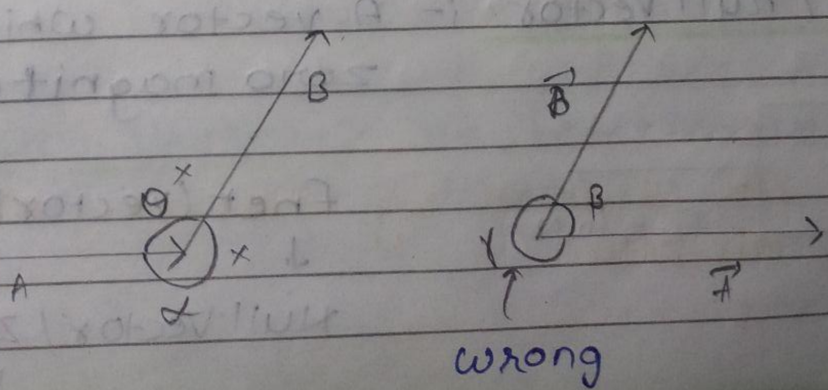


$\vec{A} = \vec{B} = \vec{C} \neq D$

But Rotation of vector will change that vector (Due to change in Direction)

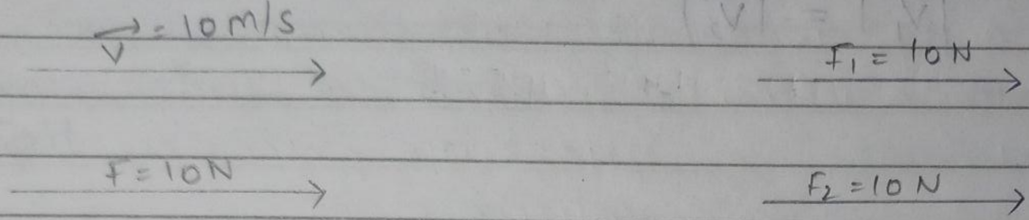
* Angle Between Vector

Angle between two vector is a smaller angle of the two angle when they are placed head to head or tail to tail



* Types of Vector

1. Equal vector :- Having same magnitude and direction of same nature

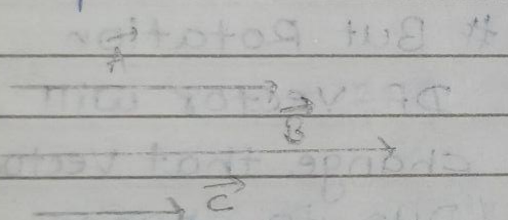


$\vec{F_1} = \vec{F_2}$

Not a equal vector

Equal Vector

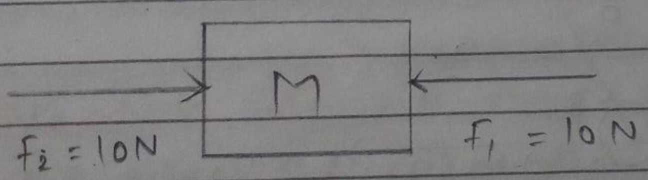
2. Parallel vector :- Having same direction but magnitude may or may not equal.



→ $\vec{A}, \vec{B}, \vec{C}$ are Parallel vectors not a equal Vector but all equal vector must be parallel

3. Anti-Parallel vector :- Vectors which have opposite direction. Magnitude may or may not be equal.

4. Zero vector / null vector :- A vector which have zero magnitude.



Fnet (vector) = 0
↓
Null Vector / Zero Vector

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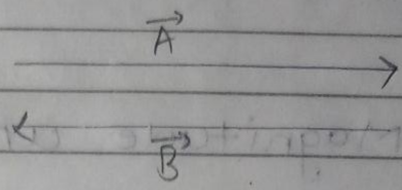
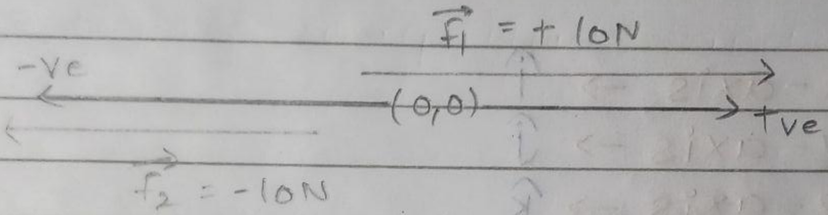
Negative Vector :- Two vectors Having same magnitude but opposite direction of same nature.

$\vec{A} = -\vec{B}$



→ In vector we will use +ve and -ve for direction



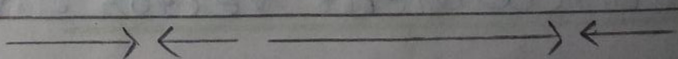
* $\vec{F_1}$ & $\vec{F_2}$ have same magnitude but opposite direction

→ Scalar :- Potential Energy

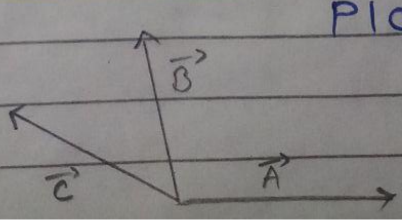
$U_1 = +5J$ $U_2 = -5J$

$$\boxed{U_1 > U_2}$$

Collinear Vector :- Vectors which are in same line.



Coplanar Vector :- vectors are in same Plane.



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Unit Vector :- Vector which have unit magnitude and represent direction.

$$\vec{A} = A\hat{A}$$

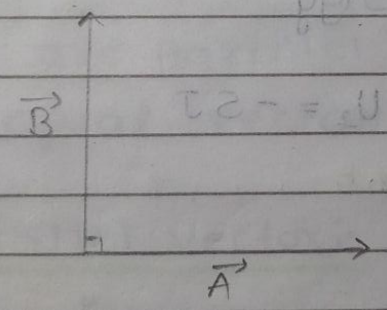
$$\hat{A} = \frac{\vec{A}}{A} = \text{Magnitude unit (1)}$$

Unit vector

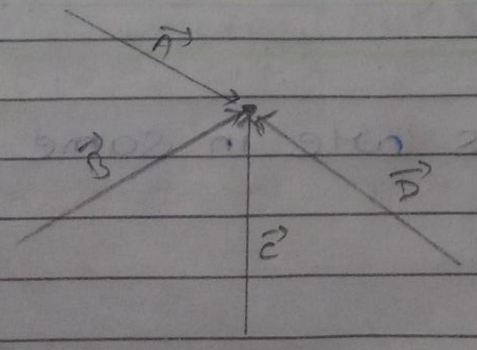
- Unit vector of x-axis → $\hat{i}$
- Unit vector of y-axis → $\hat{j}$
- Unit vector of z-axis → $\hat{k}$

~~Org~~ **orthogonal vector :-** (Perpendicular vector)

→ Vector exactly Perpendicular to each other.



Concurrent vector : vector exactly acting at a point.



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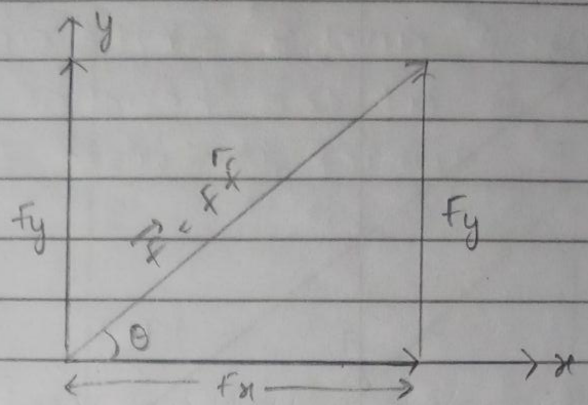
* Component of vector → Effect of vector

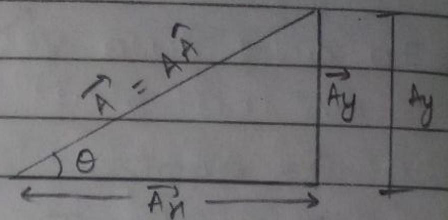


$\cos\theta = \frac{F_x}{F}$

$F_x = F\cos\theta$

$\sin\theta = \frac{F_y}{F}$

$F_y = F\sin\theta$



$\cos\theta = \frac{A_x}{A}$

$A_x = A\cos\theta$

$\sin\theta = A_y / A$

$A_y = A\sin\theta$

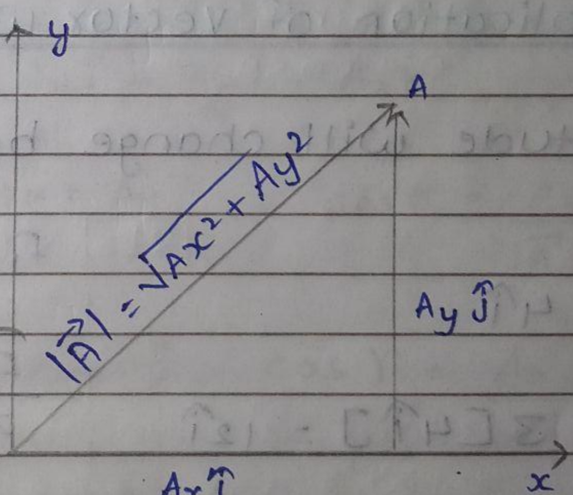
* Magnitude of Vector

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

$|\vec{A}| = \sqrt{A_x^2 + A_y^2} = A$

Magnitude of vector

Magnitude of vector



$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

→ $\vec{F_1} = 3\hat{i} - 4\hat{j}$
$\vec{F_2} = -3\hat{i} + 4\hat{j}$
} Different vector but having same magnitude.

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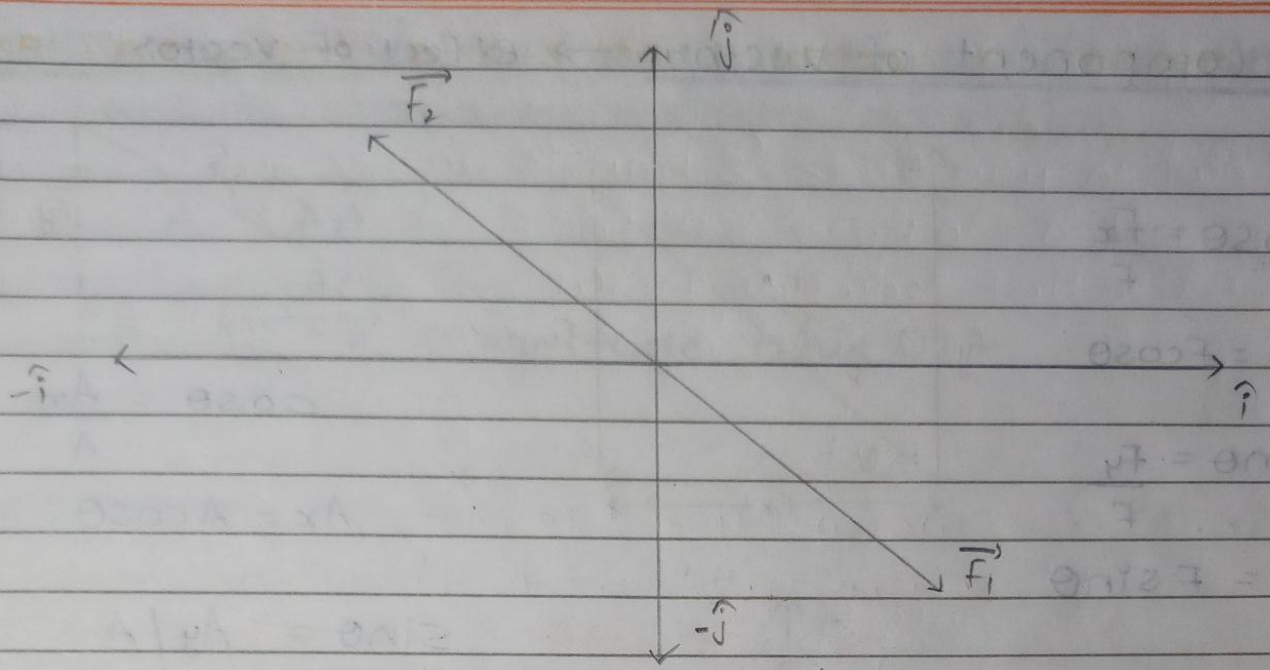


* Basic Properties of vector

1. Multiplication of vector with scalar :-

Magnitude will change but direction remains same.

$\vec{A} = 4\hat{i}$

$3\vec{A} = 3[4\hat{i}] = 12\hat{i}$

$F_1 = 4\hat{i}$ find $\frac{\vec{F_1}}{2}$

$\frac{F_1}{2} = \frac{4\hat{i}}{2} = 2\hat{i}$

2. Addition of Vector with scalar :-

5N + 4Kg → Not Possible

3. Addition of two Vector

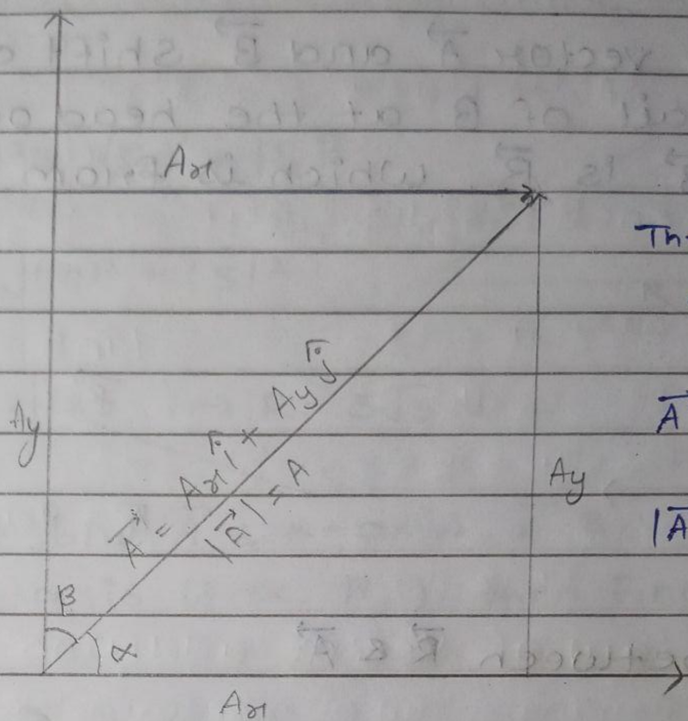
$F_1 = 3\hat{i}$, $F_2 = 4\hat{i}$, $F_{net} = (3+4)\hat{i} = 7\hat{i}$

* Direction of vector from x-axis, Y-axis and z-axis

$$\cos\alpha = \frac{A_x}{|\vec{A}|}$$

$$\tan\alpha = \frac{A_y}{A_x}$$

$$\cos\beta = \frac{A_y}{A}$$



(Cosine law)

simple triangle Theorem with direction

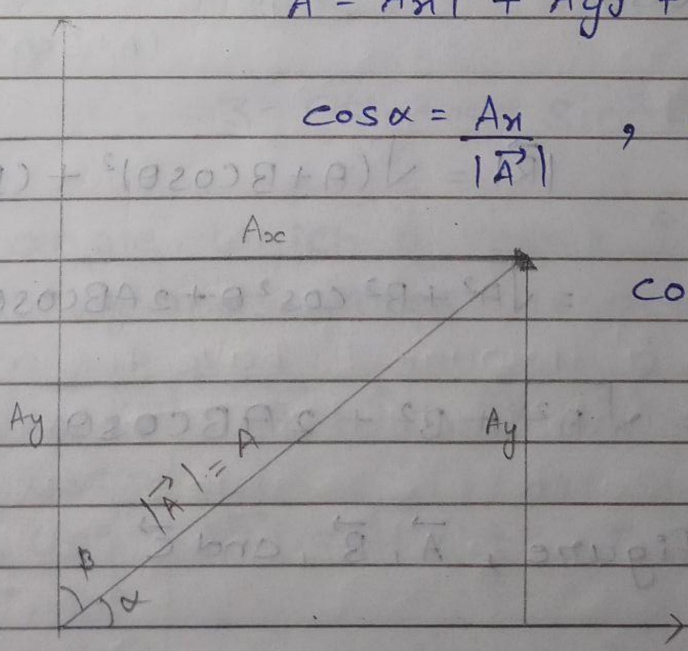
$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\cos\alpha = \frac{A_x}{|\vec{A}|}$$

$$\cos\beta = \frac{A_y}{|\vec{A}|}$$



$$\cos\alpha = \frac{A_x}{|\vec{A}|}, \quad \cos\beta = \frac{A_y}{|\vec{A}|}$$

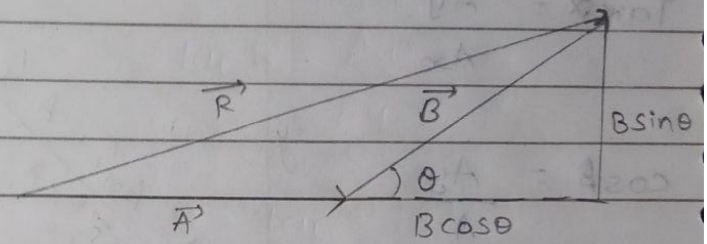
$$\cos\gamma = \frac{A_z}{|\vec{A}|}$$

Find $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = ?$

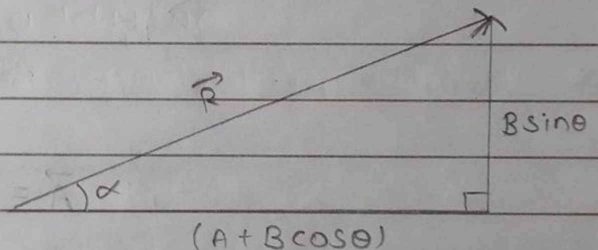
$\frac{A_x^2}{A^2} + \frac{A_y^2}{A^2} + \frac{A_z^2}{A^2} = \frac{A_x^2 + A_y^2 + A_z^2}{A^2} = 1$

* Triangle Law of vector Addition

To add two vector $\vec{A}$ and $\vec{B}$ shift any of two such that tail of B at the head of $\vec{A}$ then sum of $\vec{A}$ and $\vec{B}$ is $\vec{R}$. Which is from tail of A to head of $\vec{B}$.



α is angle between $\vec{R}$ & $\vec{A}$



* $\tan\alpha = \dfrac{B\sin\theta}{A+B\cos\theta}$

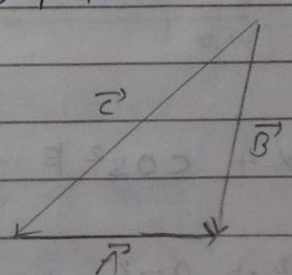
$$|\vec{R}| = \sqrt{(A+B\cos\theta)^2 + (B\sin\theta)^2}$$

$$= \sqrt{A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta}$$

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Que- for the figure, $\vec{A}$, $\vec{B}$, and $\vec{C}$

a) $\vec{A} + \vec{B} = \vec{C}$
b) $\vec{B} + \vec{C} = \vec{A}$
c) $\vec{C} + \vec{A} = \vec{B}$ ✓
d) $\vec{A} + \vec{B} + \vec{C} = 0$

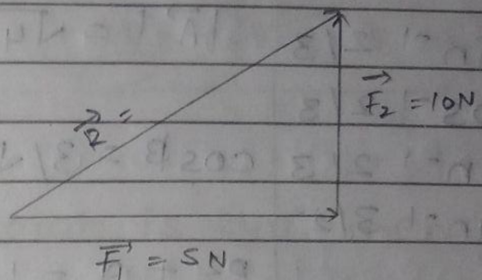


$\vec{C} + \vec{A} = \vec{B}$

Que- Two force $\vec{F_1} = 5N$ due east and $\vec{F_2} = 10N$ due north then resultant of these two forces is

a) $5\sqrt{5}N$
b) $15N$
c) $5N$
d) $\sqrt{5}N$

$(R)^2 = (F_2)^2 + (F_1)^2$



$R = \sqrt{(10)^2 + (5)^2}$

$R = \sqrt{125} \rightarrow R = 5\sqrt{5}N$

Que- If angle b/w $\vec{A}$ & x-axis, $\vec{A}$ & y-axis and $\vec{A}$ and z-axis is α, β, γ then Find $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = ?$

We know that $\rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$1-\sin^2\alpha + 1-\sin^2\beta + 1-\sin^2\gamma = 1$

$3 - [\sin^2\alpha + \sin^2\beta + \sin^2\gamma] = 1$

Que- The angle which a vector $\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ makes with x, y and z axes respectively are

a) $60°, 60°, 60°$
b) $45°, 45°, 45°$
c) $60°, 60°, 45°$
d) $45°, 45°, 60°$

$\vec{A} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$

$|\vec{A}| = \sqrt{1+1+2} = \sqrt{4} = 2$

$\cos\alpha = \frac{A_x}{A} = \frac{1}{2} = \cos\alpha = 1/2 =$
$\alpha = 60°$

$\cos\gamma = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\cos\beta = \frac{A_y}{A} = \frac{1}{2} = \beta = 60°$

$\gamma = 45°$

Que- Given $\vec{A} = 2\hat{i} + 3\hat{j}$, the angle between $\vec{A}$ and Y-axis is

a) $\sin^{-1} 2/3$

b) $\cos^{-1} 2/3$

c) $\tan^{-1} 2/3$ ✓

d) $\tan^{-1} 3/2$

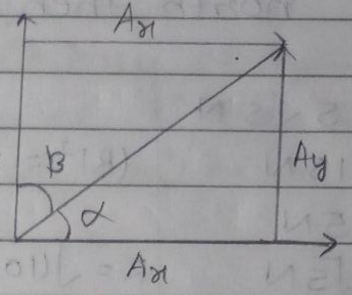
$|\vec{A}| = \sqrt{4+9} = \sqrt{13}$

$\cos\beta = 3/\sqrt{13} = B/H$

$p^2 + 9 = 13$

$p^2 = 13 - 9$

$p = \sqrt{4} = 2$

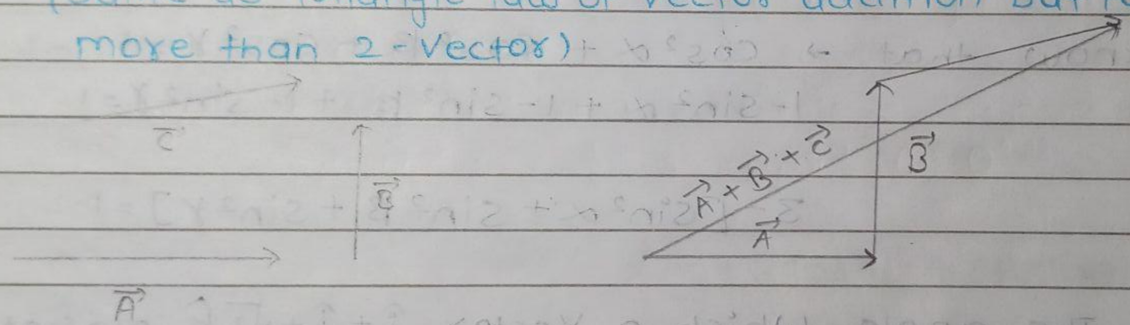


$\tan\beta = \dfrac{A_x}{A_y} = \dfrac{2}{3}$

$\beta = \tan^{-1}(2/3)$

Que- Polygon law of vector addition

(Same as Triangle law of Vector addition but for more than 2-Vector)



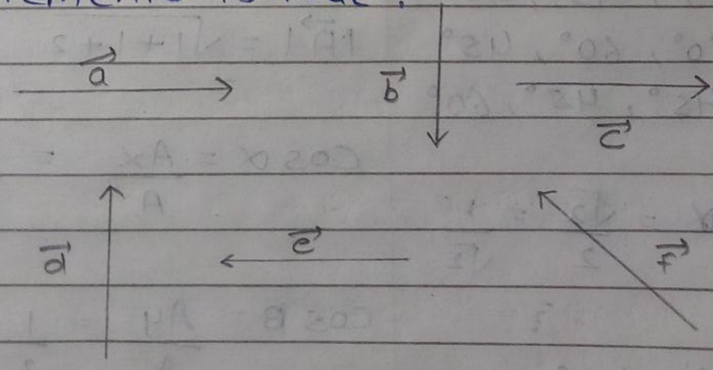
Que- Six vectors, $\vec{a}$ through $\vec{f}$ have the magnitudes and directions indicates in the figure. Which of the following statements is true?

a) $\vec{b} + \vec{c} = \vec{f}$

b) $\vec{d} + \vec{c} = \vec{f}$

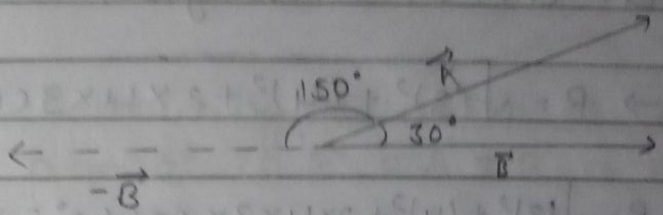
c) $\vec{d} + \vec{e} = \vec{f}$ ✓

d) $\vec{b} + \vec{e} = \vec{f}$



Que- If angle b/w $\vec{A}$ & $\vec{B}$ is 30° then angle b/w $\vec{A}$ & $-\vec{B}$ is

Ans → 150°



Que- If angle b/w two vector is 240° then result will be ?

$$R = \sqrt{A^2 + B^2 + 2AB\cos(240°)}$$

$\theta = 240° \rightarrow$ Not possible, because we have to take smallest angle.

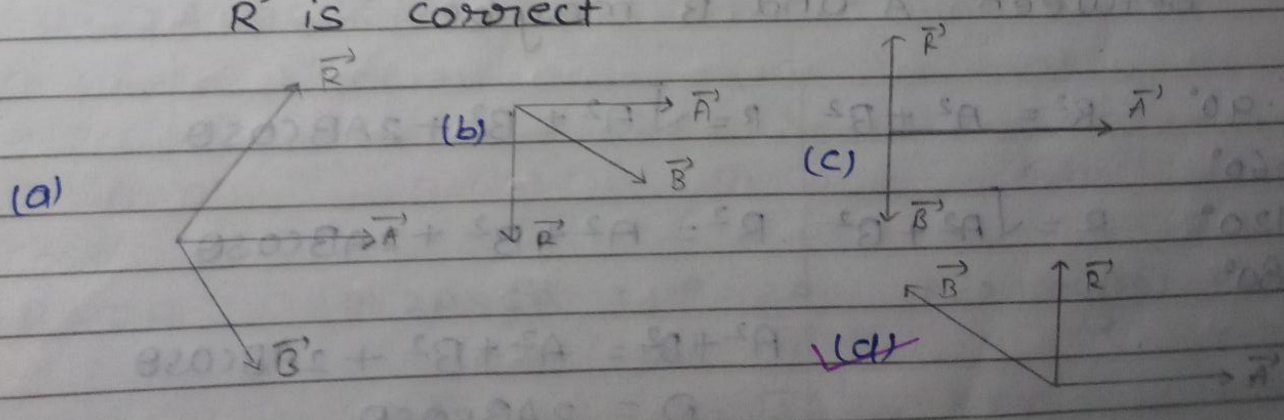
Que- If two vector of magnitude 3 and 4 then their resultant may be

(a) 1 to 7 ✓
(b) 6 to 8
(c) 2 to 7
(d) 4 to 7

$R_{max} = 3 + 4 = 7$

$R_{min} = 4 - 3 = 1$

Que- Which option relation between $\vec{A}$, $\vec{B}$ and $\vec{R}$ is correct



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Que- Two vector of magnitude 3 and 4 acting at different angle, then find Resultant

(i)- $0^\circ \rightarrow R = \sqrt{(3)^2+(4)^2+2\times4\times3\cos0^\circ} = \sqrt{9+16+24} = \sqrt{49} \Rightarrow 7$

(ii)- $60^\circ \rightarrow R = \sqrt{(3)^2+(4)^2+2\times4\times3\times\cos60^\circ} = \sqrt{9+16+12} = \sqrt{37} \approx 6.1$

(iii)- $90^\circ \rightarrow R = \sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$

(iv)- $120^\circ \rightarrow R = \sqrt{9+16+2\times4\times3\times\cos120^\circ} = \sqrt{25-12} = \sqrt{13} \approx 3.4$

(v)- $180^\circ \rightarrow R_{min} = 4-3 = 1$

Que- If $\vec{R} = \vec{A}+\vec{B} = R = A+B$ then angle b/w $\vec{A}$ and $\vec{B}$ must be

(a) 90°
(b) 60°
(c) 0° ✓
(d) 180°

$R_{max} = A+B$, at $\theta = 0^\circ$

Que- If $\vec{R} = \vec{A}+\vec{B}$ and $R^2 = A^2+B^2$ then angle between $\vec{A}$ and $\vec{B}$ may be

(a) 90° ✓
(b) 60°
(c) 120°
(d) 80°

$R^2 = A^2+B^2$

$R = \sqrt{A^2+B^2}$

$R = \sqrt{A^2+B^2+2AB\cos\theta}$

$R^2 = A^2+B^2+2AB\cos\theta$

$A^2+B^2 = A^2+B^2+2AB\cos\theta$

$0 = 2AB\cos\theta$

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$$\cos\theta = 0$$
$$\theta = 90^\circ$$

Que- Which of the following relation is correct between $\vec{A}$, $\vec{B}$ and $\vec{C}$ if $\vec{C} = \vec{A} + \vec{B}$

(a) $B+A < C < B-A$ (b) $A \leq C \geq B$

✓(c) $A-B \leq C \leq A+B$ (d) $A-B < C < A+B$

* Two vector of same magnitude acting at angle 'θ' then Resultant of the vector will be

$$|\vec{A}| = |\vec{B}| = A$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$R = \sqrt{2A^2 + 2A^2\cos\theta}$$

$$R = \sqrt{2A^2(1+\cos\theta)}$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$1 + \cos\left(\frac{2\theta}{2}\right) = 2\cos^2\left(\frac{\theta}{2}\right)$$

$$1 + \cos(\theta) = 2\cos^2(\theta/2)$$

$$R = \sqrt{2A^2(2\cos^2\theta/2)}$$

$$R = 2A\cos(\theta/2)$$

$\theta = 0^\circ$	$\theta = 60^\circ$	$\theta = 90^\circ$	$\theta = 120^\circ$	$\theta = 180^\circ$
$R = 2A$	$R = 2A \times \frac{\sqrt{3}}{2}$	$R = \frac{2A}{\sqrt{2}}$	$R = 2A\cos\left(\frac{120^\circ}{2}\right)$	$R = 2A\cos\left(\frac{180}{2}\right)$
	$R = \sqrt{3}A$	$R = \sqrt{2}A$	$R = A$	$R = 0$

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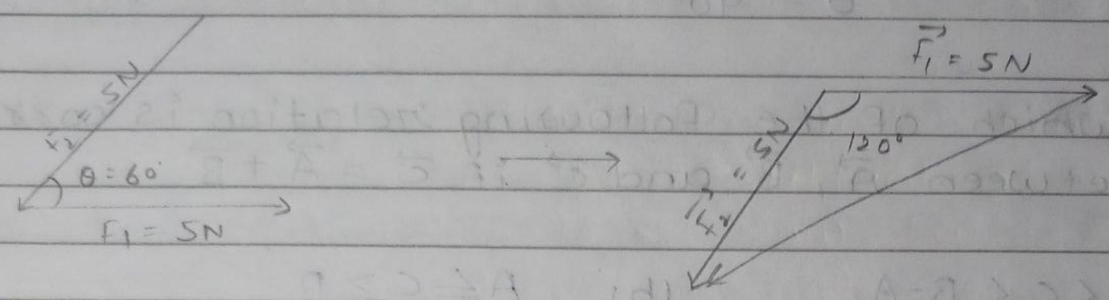
Que- Find net force $f_1 + f_2 = \vec{F}_{net} = ?$



$\vec{F}_{net} = 5N$

Que- If $\vec{R} = \vec{A} + \vec{B}$ and $|\vec{R}| = |\vec{A}| = |\vec{B}|$ then angle between $\vec{A}$ and $\vec{B}$ may be

(a) 90°
(b) 120°
(c) 60°
(d) 45°

Que- Two vector of magnitude 2 then resultant of these two vector may be ?

(a) 2	$R_{max} = 2+2 = 4$
(b) 8	
(c) 5	$R_{min} = 2-2 = 0$
(d) 6	

Que- Two vector of magnitude same A and resultant of these two vector is also A then angle between these two vector must be

(a) 45°
(b) 90°
(c) 120°
(d) 180°

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Que- Two vector of magnitude equal to each other and 10 then resultant of these two vector at 60° is.

(a) $10\sqrt{3}$ (b) 10
(c) 0 (d) 20

Que- At what angle should the two forces 2P and $\sqrt{2}P$ act so that the resultant force is $P\sqrt{10}$

(a) 45°
(b) 60°
(c) 90°
(d) 120°

$A = 2P$
$B = \sqrt{2}P$
$C = P\sqrt{10}$

$\rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$

$\rightarrow 10P^2 = 4P^2 + 2P^2 + 2(2P)\sqrt{2}P\cos\theta$

$\rightarrow (10-6) = 4\sqrt{2}\cos\theta$

$\rightarrow \sqrt{2}\cos\theta = 1$

$\rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

Que- Two forces 5N and 2N acting on the object then net Force on object must not be

(a) 2N
(b) 1N
(c) 6N
(d) Both (1) & (2)

$F_{max} = 5 + 2 = 7N$

$F_{min} = 5 - 2 = 3N$

Que- Which of the following set of three force may be in equilibrium [Net force zero]

(a) 3N, 5N, 1N
(b) 3N, 5N, 6N

(c) 3N, 5N, 9N $\qquad \vec{F_1} + \vec{F_2} + \vec{F_3} = 0$
(d) 3N, 5N, 16N $\qquad \vec{F_1} + \vec{F_2} = -\vec{F_3}$

Que- The Ratio of maximum and minimum magnitudes of the resultant of two vectors $\vec{a}$ and $\vec{b}$ is 3:1 Now, $|\vec{a}|$ is equal to:

(a) $|\vec{b}|$
✓(b) $2|\vec{b}|$
(c) $3|\vec{b}|$
(d) $4|\vec{b}|$

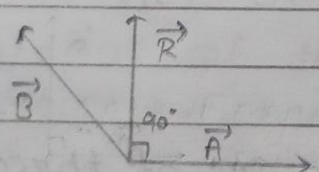
$$\frac{a+b}{a-b} = \frac{3}{1} \Rightarrow a+b = 3a-3b$$
$$\Rightarrow 4b = 2a$$
$$\Rightarrow 2b = a$$

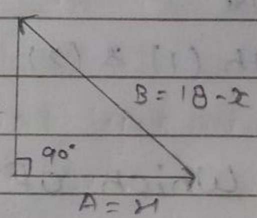
Que- Four Forces of magnitude P, 2P, 3P and 4P act along the four sides of a square ABCD in cyclic order. Find the resultant force.

(a) 2P $\qquad$ (b) $3\sqrt{2}$ P $\qquad$ (c) 0 $\qquad$ ✓(d) $2\sqrt{2}$ P

Que- The sum of the magnitudes of the two forces acting at point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the magnitudes of forces?

(a) 12, 5
(b) 14, 4
✓(c) 5, 13
(d) 10, 8





$|\vec{A}+\vec{B}| = |\vec{R}| = 12$

$$\rightarrow A^2 + R^2 = B^2$$
$$x^2 + (12)^2 = (18-x)^2$$
$$x^2 + 144 = 324 + x^2 - 36x$$
$$x = 5$$

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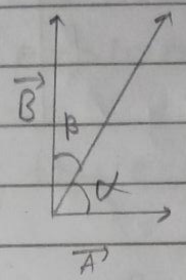
Que- The resultant of $\vec{A}$ and $\vec{B}$ makes an angle α with $\vec{A}$ and β with $\vec{B}$ then

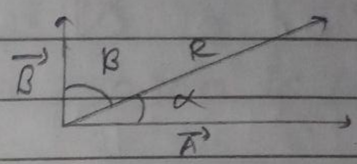
(a) $\alpha < \beta$

~~(b)~~ $\alpha < \beta$ if $A > B$

(c) $\alpha > \beta$

(d) $\alpha = \beta$





$B > A$ ($\alpha > \beta$) $\qquad$ $A > B \rightarrow (\alpha < \beta)$

* Resultant of vector addition must be in the Plane of two vector → True

→ can sum of three non co-Planar vector is zero?

= No → Not Possible

Minimum no. of vector whose result can be zero

→ Two but they must be equal and Opposite

Minimum no. of unequal vector whose result can be zero

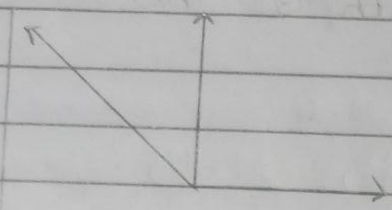
→ Three

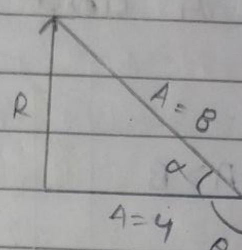
Minimum no. of Non-coplanar vector whose result can be zero is

→ Four

Que- The sum of vectors ($A = 4$) units and $B = 8$ units is perpendicular to A. The angle θ between A and B is

(a) 60°
(b) 120°
(c) 135°
(d) 145°





$\theta = 180 - 60^\circ = 120^\circ$

$\cos\alpha = 4/8$

$\cos\alpha = 1/2$

$\alpha = 60^\circ$

Que- Assertion- Minimum no. of vectors having unequal magnitude in a plane required to give zero resultant is three

Reason- If vector addition of three vectors is zero, then they must lie in a plane

(a) If both Assertion & Reason are true & the reason is a correct explanation of the Assertion.

(b) If both Assertion & Reason are true but Reason is not a correct explanation of the Assertion.

(c) If Assertion is True but the reason is false

(d) If both Assertion & Reason are false.

Que- Assertion - Any physical Quantity having direction is a vector.

Reason- The vector require both magnitude as well as direction

(a) If both Assertion & Reason are true & the reason is a correct explanation of the assertion.

(b) If both Assertion & Reason are true but reason is not a correct explanation of the assertion.

(c) If assertion is true but the reason is false.

(d) If both Assertion & Reason are false.

Que- Assertion- A Physical Quantity can be regarded as a vector, if magnitude as well as direction is associated with it

Reason- A Physical Quantity can be regarded as a scalar Quantity. if it is associated with magnitude only.

(a) If both Assertion & Reason are True & the reason is a correct explanation of the Assertion.

(b) If both Assertion & Reason are True but reason is not a correct explanation of the assertion.

(c) If Assertion is True but the Reason is false.

(d) If both Assertion & Reason are false

जिसके पास सिर्फ Magnitude हो वह scalar हो सकता है
↳ Wrong.

जिसके पास Magnitude direction दोनो है, वह vector हो सकता है - सही है

Ex- current

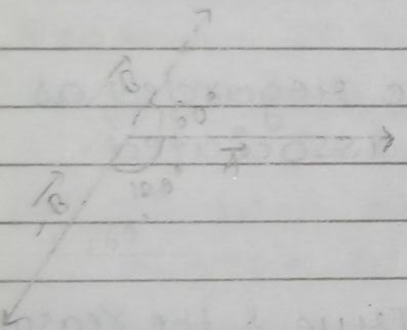
Que- If $\vec{A} = 2\hat{i} + \sqrt{5}\hat{j}$ and $\vec{B} = 5\hat{i} + \sqrt{5}\hat{j}$ then find a vector which is parallel to $\vec{A}$ and magnitude equal to $\vec{B}$

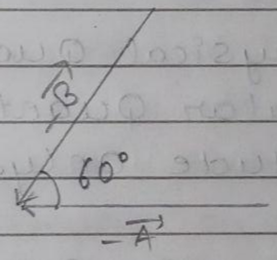
Direction of $\vec{C}$ is along $\vec{A}$

$$|\vec{C}| = |\vec{B}|$$

Que- If angle b/w $\vec{A}$ & $\vec{B}$ is 60° then find angle b/w

(i) $\vec{A}$ & $-\vec{B}$ - 120° (ii) $-\vec{A}$ & $-\vec{B}$ - 60°

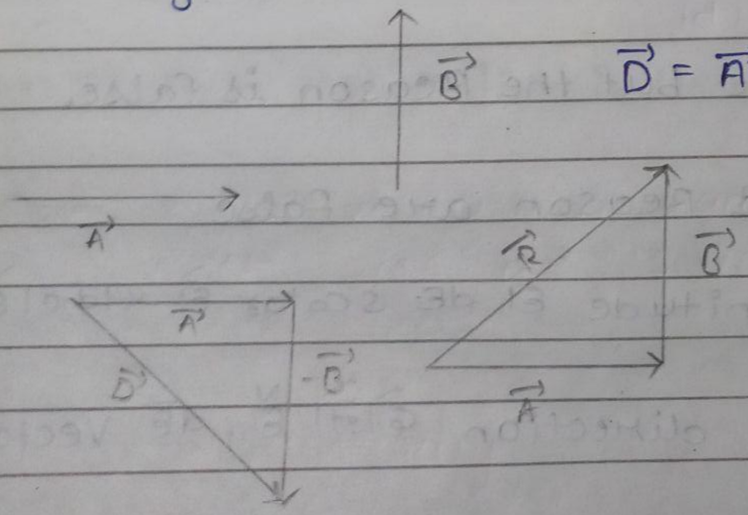


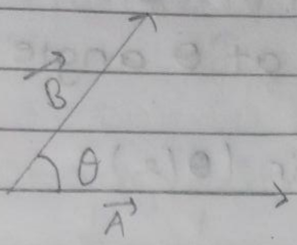


* Vector subtraction

Triangle law of vector subtraction होता ही नहीं

$$\vec{D} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

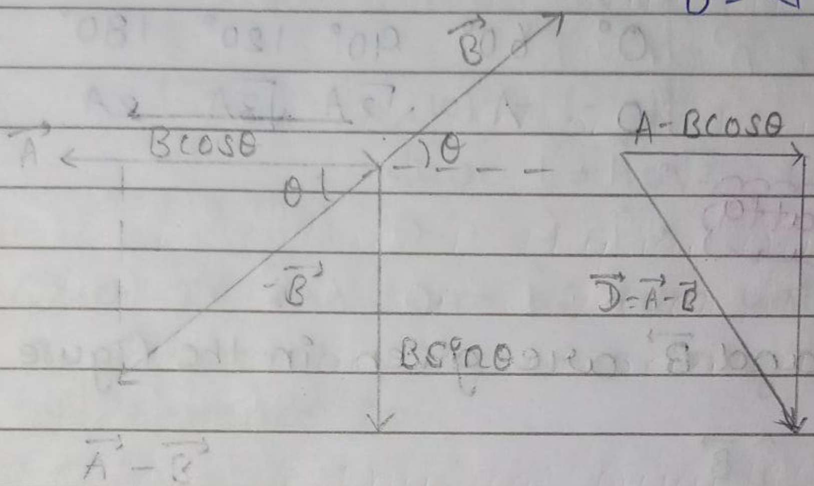




$$|\vec{D}| = \sqrt{(A - B\cos\theta)^2 + (B\sin\theta)^2}$$

$$= \sqrt{A^2 + B^2\cos^2\theta - 2AB\cos\theta + B^2\sin^2\theta}$$

$$D = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$



Que- If angle b/w $\vec{A}$ & $\vec{B}$ is θ, then Angle b/w $\vec{A}$ & $-\vec{B}$ is $(180 - \theta)$

$\vec{D} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ $\quad \cos(180 - \theta) = -\cos\theta$

$$D = \sqrt{A^2 + B^2 + 2AB\cos(180 - \theta)}$$

$$D = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$\theta = 0^\circ$ $\quad$ $\theta = 90^\circ$ $\quad$ $\theta = 180^\circ \rightarrow \cos 180^\circ = -1$

$D_{min} = A - B$ $\quad$ $D = \sqrt{A^2 + B^2}$ $\quad$ $D_{max} = A + B$

If two vector of same magnitude $|\vec{A}| = |\vec{B}| = A$ then find magnitude of difference at θ angle

$$D = \sqrt{A^2 + A^2 - 2A^2\cos\theta}$$

$$D = \sqrt{2A^2(1-\cos\theta)}$$

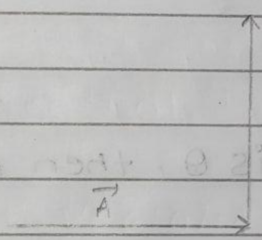
$$D = \sqrt{2A^2(2\sin^2(\theta/2))}$$

$$D = 2A\sin(\theta/2)$$

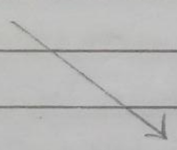
$D = 2A\sin(\theta/2)$

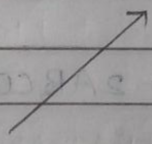
0°	60°	90°	120°	180°
0	A	$\sqrt{2}A$	$\sqrt{3}A$	$2A$

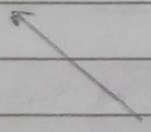
Que- Two vector $\vec{A}$ and $\vec{B}$ are given in the figure



Then $\vec{A} - \vec{B}$ is given by

✓(a) 

(b) 

(c) 

(d) None of these

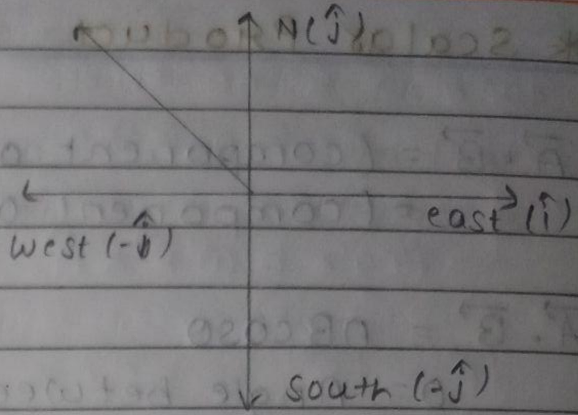
Que- initial velocity of Ramlal is 10 m/s in east after sometime its velocity becomes 10 m/s north then find.

(i) change in velocity

(ii) find magnitude of change in velocity.

$V_i = 10\hat{i}$ (east)

$V_f = 10\hat{j}$ (North)



Change in velocity $= \vec{V_f} - \vec{V_i}$

$$\vec{\Delta V} = 10\hat{j} - 10\hat{i}$$

$$|\vec{\Delta V}| = \sqrt{(10)^2 + (-10)^2}$$

$$= \sqrt{200} = 10\sqrt{2} \text{ (North west)}$$

Que- If the sum of two units vectors is a unit vector, then the magnitude of their difference is

(a) $\sqrt{3}$ $\qquad |\vec{A}| = |\vec{B}| = 1$, $\theta = 120°$

(b) $\sqrt{2}$

(c) $\sqrt{5}$ $\qquad |\vec{R}| = 1$

(d) $1/\sqrt{2}$ $\qquad |\vec{D}| = \sqrt{3}A = \sqrt{3} \times 1 = \sqrt{3}$

Que- $\vec{A} + \vec{B} = \vec{R}$ and $\vec{A} - \vec{B} = \vec{D}$ then find angle between $\vec{A}$ and $\vec{B}$ if $R = D$

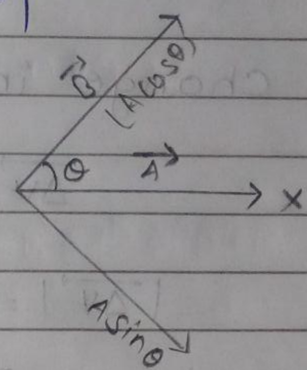
$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$D = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

* Scalar Product [Dot Product of vector]

$\vec{A}\cdot\vec{B} = $ (component of A along B) $\times |\vec{B}|$
= (component of B along A) $\times$ A



$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\theta =$ Angle between $\vec{A}$ and $\vec{B}$

- $\hat{i}\cdot\hat{i} = 1\times1\cos0^\circ = 1$
- $\hat{j}\cdot\hat{j} = 1\times1\cos0^\circ = 1$
- $\hat{k}\cdot\hat{k} = 1\times1\cos0^\circ = 1$

- $\hat{i}\cdot\hat{j} = 0$
- $\hat{i}\cdot\hat{k} = 0$
- $\hat{k}\cdot\hat{j} = 0$

comp. of B along A

$\vec{A}\cdot\vec{B} = AB\cos\theta$

comp. of A along B

* Application of Dot Product

(i) Angle b/w vectors

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

Angle b/w $\vec{A}$ & $\vec{B}$

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$\vec{A}\cdot\vec{B} = (A_xB_x + A_yB_y + A_zB_z)$

(ii) To find unit vector

If $\vec{A}$ is a unit vector, then

$\vec{A}\cdot\vec{A} = 1\cdot1\cos0^\circ = 1$

$\vec{A}\cdot\vec{A} = 1$

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(iii) To check perpendicular $\vec{A}$ and $\vec{B}$

If dot product of two vector is zero then that two vector must be perpendicular (orthogonal)

$$\vec{A}\cdot\vec{B} = AB\cos 90^\circ = 0$$

A Perpendicular to B

Que - If $\vec{A} = 2\hat{i} + 3\hat{j} - a\hat{k}$ and $B = \hat{i} - 2\hat{j} + 4\hat{k}$. find α. If $\vec{A}$ is $\perp$ to $\vec{B}$.

$$\vec{A}\cdot\vec{B} = 0$$

$$2 - 6 + 4\alpha = 0$$

$$-4 + 4\alpha = 0$$

$$4\alpha = 4$$

$$\alpha = \frac{\cancel{4}}{\cancel{4}} = 1$$

Que- If $\vec{A} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ and $\vec{B} = 4\hat{i}$. Find angle between $\vec{A}$ and $\vec{B}$

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$\vec{R} = \vec{A} + \vec{B}$ → In a same Plane Of $\vec{A}$ and $\vec{B}$	$\vec{D} = \vec{A} - \vec{B}$ → In a Plane of $\vec{A}$ and $\vec{B}$
$\lvert\vec{R}\rvert = \sqrt{A^2 + B^2 + 2AB\cos\theta}$	$D = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

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$\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$

→ Angle b/w $\vec{R}$ & $\vec{A}$

$|\vec{A}| = |\vec{B}|$

→ $D = 2A\sin(\theta/2)$

$|\vec{A}| = |\vec{B}|$

$\boxed{A - B \leq |\vec{R}| \leq A + B}$

$A - B \leq D \leq A + B$

$R = 2A\cos(\theta/2)$

0°	60°	90°	120°	180°
R = 2A	$\sqrt{3}A$	$\sqrt{2}A$	A	0

Que- If $\vec{A} = 2\hat{i} + \sqrt{5}\hat{j}$ and $\vec{B} = 5\hat{i} + \sqrt{5}\hat{j}$ then find a vector which is parallel of $\vec{A}$ and magnitude equal to $\vec{B}$

Let that vector is $\vec{C}$

$|\vec{B}| = \sqrt{(5)^2 + (\sqrt{5})^2}$

$= \sqrt{25 + 5}$

$= \sqrt{30}$

$|\vec{C}| = |\vec{B}|$

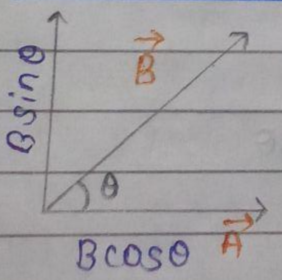
$\hat{C} = \hat{A}$

$\hat{A} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{2\hat{i} + \sqrt{5}\hat{j}}{\sqrt{(2)^2 + (\sqrt{5})^2}} = \dfrac{2\hat{i} + \sqrt{5}\hat{j}}{3}$

$\vec{C} = |\vec{B}|\hat{A}$

$= \sqrt{30}\left[\dfrac{2\hat{i} + \sqrt{5}\hat{j}}{3}\right]$

Dot Product (Scalar Product)



$\vec{A} \cdot \vec{B} = AB\cos\theta$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$\vec{A}\cdot\vec{B}_{max} = AB$ At $\theta = 0°$

at $90°$ $\vec{A}\cdot\vec{B} = 0$

at $180°$ $\vec{A}\cdot\vec{B} = -AB$

$0° \le \theta < 90°$ $\vec{A}\cdot\vec{B} = +ve$

$90° < \theta \le 180°$ $\vec{A}\cdot\vec{B} = -ve$

* Application of Dot Product

1. To find Angle b/w vector

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

2. To find orthogonal vector

$\vec{A}\cdot\vec{B} = 0$

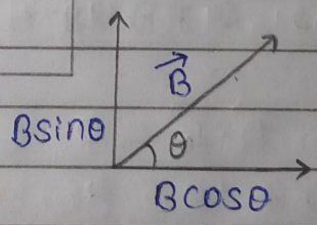
$\rightarrow \underbrace{\vec{A} \perp_r \vec{B}}$

3. To check unit vector

Let $\vec{A}$ is a unit vector

$\vec{A}\cdot\vec{A} = 1.1\cos 0° = 1$

4. To take component of one vector along other



$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = BA\cos\theta$

$$B\cos\theta = \frac{\vec{A}\cdot\vec{B}}{A}$$

$\downarrow$

component of B along A

$$A\cos\theta = \frac{\vec{A}\cdot\vec{B}}{B}$$

$\downarrow$

component of A along B

Que- If $|\vec{A}| = 2$ and $|\vec{B}| = 4$, $\theta = 60°$ then find $\vec{A}\cdot\vec{B}$

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$$\vec{A}\cdot\vec{B} = AB\cos\theta$$
$$= 2\times 4\times \frac{1}{2}$$
$$= 4$$

Que- If $\vec{A} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j}$ then prove that $\vec{A}$ is a unit vector

$$\vec{A}\cdot\vec{A} = 1$$

$|\vec{A}| = 1$

$$(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})\cdot(\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) = 1$$

$\sqrt{\sin^2\theta + \cos^2\theta} = 1$

$$\sin^2\theta + \cos^2\theta = 1$$
$$[1 = 1]$$

Que- If $\vec{A} = 0.5\,\hat{i} + 0.4\,\hat{j} - \alpha\hat{k}$ then find α if $\vec{A}$ is unit vector

$\vec{A}\cdot\vec{A} = 1$

$0.25 + 0.16 + \alpha^2 = 1$

$|\vec{A}| = 1$

$\alpha^2 = 1 - 0.25 - 0.16$

$\sqrt{(0.5)^2 + (0.4)^2 + \alpha^2} = 1$

$\left(\frac{5}{10}\right)^2 + \left(\frac{4}{10}\right)^2 + \alpha^2 = 1^2$

Que- If $\vec{A} = 2\hat{i} + 3\hat{j} - \alpha\hat{k}$ and $\vec{B} = \hat{i} - 2\hat{j} + 4\hat{k}$. find α. If $\vec{A}$ is perpendicular to $\vec{B}$

$\vec{A}\cdot\vec{B} = 0$

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$(2\hat{i}+3\hat{j}-\alpha\hat{k})\cdot(\hat{i}-2\hat{j}+4\hat{k})=0$

$2-6-4\alpha=0$

$-4-4\alpha=0$

$4\alpha=-4$

$\alpha=-1$

Que- If $\vec{A}=2\hat{i}+2\hat{j}$ and $\vec{B}=-2\hat{i}+2\hat{j}$ then find angle b/w $\vec{A}$ and $\vec{B}$

$\vec{A}\cdot\vec{B}=AB\cos\theta$

$\cos\theta=\dfrac{\vec{A}\cdot\vec{B}}{AB}=\dfrac{-4+4}{AB}=0$

Que- If $\vec{A}=2\hat{i}+6\hat{j}+3\hat{k}$ and $\vec{B}=4\hat{i}$. Find angle b/w $\vec{A}$ and $\vec{B}$

$\vec{B}=4\hat{i}$

$\vec{A}\cdot\vec{B}=AB\cos\theta$ $\qquad |\vec{B}|=4$

$8=\sqrt{2^2+6^2+3^2}\,\sqrt{4^2}\cos\theta$

$\cancel{8}=\sqrt{4+36+9}\times\cancel{4}\cos\theta$

$2=7\cos\theta$

$\cos\theta=\dfrac{2}{7}$ $\qquad \theta=\cos^{-1}\left(\dfrac{2}{7}\right)$

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Que- If vector $\vec{A} = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$ and $B = \cos \omega t/2\,\hat{i} + \sin \omega t/2\,\hat{j}$ are function of time, then the value of t at which they are orthogonal to each other is

(a) $t = \pi/\omega$

b) $t = 0$

c) $t = \pi/4\omega$

d) $t = \pi/2\omega$

$$\vec{A}\cdot\vec{B} = 0$$

$$(\cos(\omega t)\hat{i} + \sin(\omega t)\hat{j})\cdot\left(\cos\left(\frac{\omega t}{2}\right)\hat{i} + \sin\left(\frac{\omega t}{2}\right)\hat{j}\right) = 0$$

$$\cos(\omega t)\cdot\cos\left(\frac{\omega t}{2}\right) + \sin(\omega t)\sin\left(\frac{\omega t}{2}\right) = 0$$

$$\cos\left(\omega t - \frac{\omega t}{2}\right) = 0, \quad \cos\left(\frac{\omega t}{2}\right) = 0$$

$$\frac{\omega t}{\cancel{2}} = \left(\frac{\pi}{\cancel{2}}\right) \qquad t = \frac{\pi}{\omega}$$

Que- If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$, then the value of α is

a) 1/2

(b) -1/2

c) 1

d) -1

$$\vec{A}\cdot\vec{B} = 0$$

$$-8 + 12 + 8\alpha = 0$$

$$4 + 8\alpha = 0$$

$$8\alpha = -4$$

$$\alpha = -\frac{4}{8} \rightarrow -\frac{1}{2}$$

Que- The vector sum of two forces is perpendicular to their vector differences. In that case, the forces

a) are equal to each other
b) are equal to each other in magnitude.
c) are not equal to each other in magnitude
d) cannot be Predicted.

Que- The angle b/w the two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $B = 3\hat{i} + 4\hat{j} - 5\hat{k}$ will be

a) 90°
b) 180°
c) zero
d) 45°

$\vec{A} \cdot \vec{B} = AB\cos\theta$

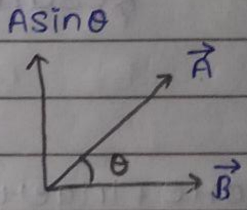
$(3\hat{i} + 4\hat{j} + 5\hat{k}) \cdot (3\hat{i} + 4\hat{j} - 5\hat{k}) = \sqrt{50} \times \sqrt{50}\cos\theta$

$\cancel{9 + 16 - 25}$

$0 = 50\cos\theta$

CROSS PRODUCT

[Vector - Product]



(vector) × (vector) = vector

$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$

= (component of A Perpendicular to B) B

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta\,\hat{n}$ → dirn of ($\vec{A}$ and $\vec{B}$)

Direction of $\vec{A} \times \vec{B}$ is Perpendicular to $\vec{A}$ & $\vec{B}$

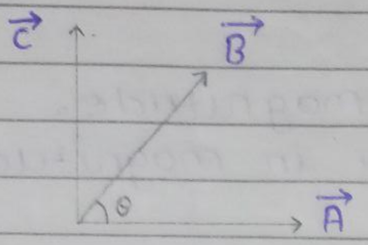
$\vec{C} = \vec{A} \times \vec{B}$ → Angle b/w $\vec{C}$ & $\vec{A}$ is

$\vec{C} \perp r\ \vec{A}$ → $\theta = 90°$

$\vec{C} \perp r\ \vec{B}$

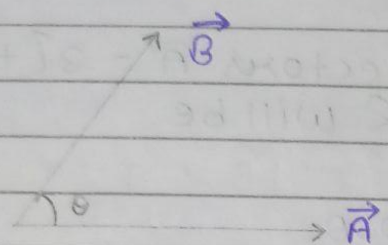
$\vec{C} \cdot \vec{A} = 0$

$\vec{C} \cdot \vec{B} = 0$



$\vec{C} = \vec{A} \times \vec{B}$

↳ $\vec{C}$ is $\perp r$ to the Plane of $\vec{A}$ & $\vec{B}$



$\vec{A} \times \vec{B} = \odot$

first vector   second vector

Director of $\vec{A} \times \vec{B}$

$\vec{R} = \vec{A} \times \vec{B}$

Result   1st Vector   2nd Vector

Place forefinger in the director of 1st vector and then slap in the direction of $\vec{B}$ (2nd vector) then thumb represent direction of $(\vec{A} \times \vec{B})$

Que- If $\vec{A}$ is in → east and $\vec{B}$ is in north then $\vec{A} \times \vec{B}$ will be in

Outward

Que- Direction of $\vec{A} \times \vec{B}$ always Perpendicular to the $\vec{A}$ and $\vec{B}$

$(\vec{A} \times \vec{B}) \perp$ to $\vec{A}$

$(\vec{A} \times \vec{B}) \cdot \vec{A} = 0$

$\underbrace{(\vec{A} \times \vec{B})}_{\perp r \text{ to } B} \cdot \vec{B} = 0$

Que- $\vec{A}$ $\odot$ outward

1. $\longrightarrow \vec{B}$

$\vec{A} \times \vec{B} = \uparrow$

2. $\vec{A}$ $\rightarrow$, $\vec{B}$ $\downarrow$

$\vec{A} \times \vec{B}$ $\otimes$

3. $\vec{A}$ $\otimes$

$\vec{B}$ $\leftarrow$

$\vec{B} \times \vec{A} = \downarrow$

4. B $\otimes$

$\leftarrow \vec{A}$

$\vec{A} \times \vec{B} = \downarrow$

Que- If $\vec{A} \times \vec{B} = \sqrt{3}(\vec{A} \cdot \vec{B})$ then find angle between $\vec{A}$ and $\vec{B}$

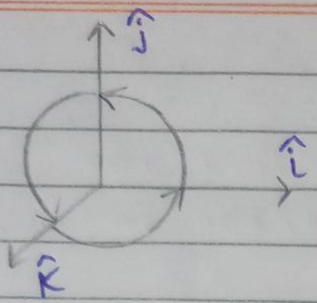
$\cancel{AB}\sin\theta = \sqrt{3}\,\cancel{AB}\cos\theta$

$\rightarrow \sin\theta / \cos\theta = \sqrt{3}$

$\rightarrow \tan\theta = \sqrt{3}$

$\rightarrow \theta = 60^\circ$

• $\hat{i} \times \hat{i} = 1 \times 1 \times \sin 0^\circ = 0$	• $\hat{i} \cdot \hat{i} = 1$
• $\hat{j} \times \hat{j} = 0$	• $\hat{j} \cdot \hat{j} = 1$
• $\hat{k} \times \hat{k} = 0$	• $\hat{k} \cdot \hat{k} = 1$



$\hat{i} \times \hat{j} = +\hat{k}$

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{i} \times \hat{j} = +\hat{k}$

$\hat{k} \times \hat{i} = +\hat{j}$

$\hat{j} \times \hat{k} = +\hat{i}$

$\hat{i} \times \hat{k} = -\hat{j}$

Que- If $\vec{A} = 2\hat{i} - 2\hat{j}$ and $\vec{B} = 5\hat{k}$ then find $\vec{A} \times \vec{B}$

$$\vec{A} \times \vec{B} = (2\hat{i} - 2\hat{j}) \times 5\hat{k}$$
$$= -10\hat{j} - 10\hat{i}$$
$$= -10[\hat{i} + \hat{j}]$$

Que- If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ & $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\Rightarrow \hat{i}|A_yB_z - A_zB_y| - \hat{j}|A_xB_z - A_zB_x| + \hat{k}|A_xB_y - A_yB_x|$

$\Rightarrow \hat{i}|A_yB_z - A_zB_y| + \hat{j}|A_zB_x - A_zB_z| + \hat{k}|A_xB_y - A_yB_x|$

Que- $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ $\qquad \vec{B} = 2\hat{i} + \hat{j} - 3\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & -3 \end{vmatrix}$$

$\Rightarrow \hat{i}(-9+1) - \hat{j}(-6+2) + \hat{k}(2-6)$

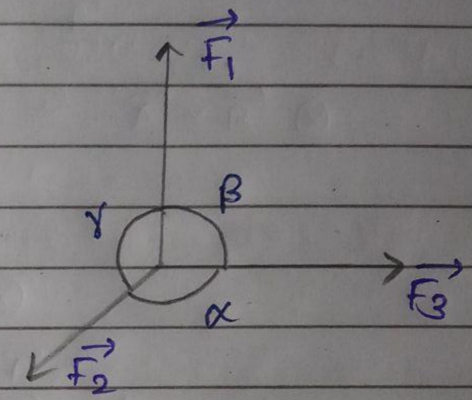
$-8\hat{i}+4\hat{j}-4\hat{k}$

LAMIS Theorem

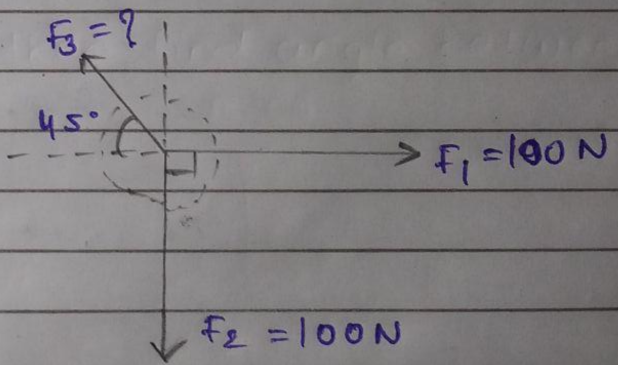
If vector sum of three force is zero, hence relation between them

If $\vec{F_1}+\vec{F_2}+\vec{F_3}=0$

$$\frac{|\vec{F}|}{\sin\alpha}=\frac{|\vec{F_2}|}{\sin\beta}=\frac{|\vec{F_3}|}{\sin\gamma}$$



Que- find value of F_3 ?



$$\frac{F_3}{\sin 90°}=\frac{F_1}{\sin 135°}$$

$$F_3=\frac{F_1}{1/\sqrt{2}}$$

$$F_3=100\sqrt{2}$$